Flight Envelope Protections using Phase Plane Limits and Backstepping Control

Rasmus Steffensen*, Agnes Gabrys*, Florian Holzapfel

Abstract This paper introduces a new way to enforce aircraft flight envelope protections based on specifications in the phase plane of a protected variable. The protection is based on applying backstepping on a set of transformed coordinates, such that accurate phase plane tracking can be achieved. Piecewise polynomials are proposed for the limit in the phase plane, to obtain less conservative protections that enhance the aircraft performance around the protected variables. The approach has a well defined and intuitive response behaviour close to the limit in which the distance from the limit is defined with a constant time to violation. Above the limit, aggressive disturbance rejection can be achieved. The results are shown in simulation of a multi engine general aviation aircraft.

1 Introduction

Aircraft loss of control in flight, LOC-I, is one of the largest contributors to aviation accidents today. In the period 2006-2015, there have been 15 fatal accidents in commercial jet aviation directly related to LOC-I with fatalities exceeding 1300 persons [2]. In general aviation, there were almost 200 fatal accidents related to loss of control during the period 2011-2015 according to [3]. Developing techniques to prevent these types of accidents is an important task within the domain of flight safety of both modern airliners, but also general aviation aircraft and remotely piloted ve-

Rasmus Steffensen
Technical University of Munich, Bolzmannstrasse 15, e-mail: rasmus.steffensen@tum.de

Agnes Gabrys
Technical University of Munich, Bolzmannstrasse 15 e-mail: agnes.gabrys@tum.de

Florian Holzapfel
Technical University of Munich, Bolzmannstrasse 15 e-mail: florian.holzapfel@tum.de

*These authors contributed equally to this work

hicles. One method to prevent these accidents is the incorporation of reliable and non-conservative flight envelope protection systems.

The purpose of a flight envelope protection is to prevent an aircraft to escape the safe envelope, i.e. from exceeding the boundary of some protected parameter. In modern aviation, the most typical protections include angle of attack, load factor, pitch angle, bank angle and airspeed. Protections are often incorporated by a combination of visual means, haptic feedback in the control inceptor and/or by soft or hard limitations invoked in the flight control system. One important aspect of envelope protections deals with the task of shaping the response of the aircraft close to the limit of a protected variable such as the angle of attack or load factor. The incorporation of this response into the flight control system is non-trivial, and many possibilities exist to apply the protection control law within the flight control system. Some of the methods include limiting of commands to a nominal controller, limiting or adding corrections to the control signal from the nominal controller, employing an integrated constrained control law, using pseudo control limitations and limiting directly in the reference model for a nominal controller.

Limiting the control variable after the nominal controller, control limiting, can be achieved with different kind of control techniques. In [4], control limiting is performed with PID controllers to track the different envelope limits. When the control signal from the nominal controller exceeds the output from the limiting controller, a switching logic engages the respective protection PID controller to track the protected variable. In [8], a correction to the control signal from the nominal controller is calculated based on a safe response profile for the limited parameter. The response profile is inspired by obstacle avoidance techniques from robotics, considering the limit as an obstacle to be avoided. Another strategy is limiting commands to the nominal controller, command limitation, which prevents commands that would result in exceedance of the protected variable to be send to the nominal controller. In [4], this is performed using parameter projections, to limit the attitude rate commanded to a RCAH, Rate Command Attitude Hold control system. Another method within the domain of envelope protections are integrated constrained control laws which employ the envelope protection directly in the control law by imposing constraints on the allowed output variable. In [4] Model Predictive Control which has the possibility to include both input and output constraints in the control signal calculation, is combined with an INDI, Incremental Nonlinear Dynamic Inversion controller, to satisfy the envelope limits. The linearised system, coming from the feedback linearisation is controlled using the MPC scheme, and incorporates the constraints on the linearised coordinates of the INDI. In [7], a backstepping control law is constructed for the longitudinal dynamics of an aircraft, and by using a barrier control Lyapunov function, this enforces the limits on the output, by preventing the output state to enter the protected region. Pseudo/virtual control limits can in the case of a nominal controller of the NDI type, be employed to limit the output variables. Different techniques to compute the pseudo control limits can be used, in [4], parameter projection is used to impose the limits. In [10] a Dynamic Trim concept is considered as pilot cueing for manual envelope protection. It separates slow and fast dynamics, and thereby can be used to associate control limits to the expected steady state response. This technique does not consider the transient response, and is hence only suitable for steady state critical variables. In [1],[11], [12], a protection is employed in the reference model of different control strategies, and is based on considerations in the phase plane of the protected variable.

In this paper, an envelope protection system with a protection domain in the phase plane of the protected variable is considered. Backstepping control, see [6], is used as guidance for the construction of the protection control law with appropriate assumptions and approximations. An advantage of this approach is well defined response dynamics close to the boundary, but still having the possibility to be non-conservative further away from the protection boundary. The approach considers also the transient behaviour of the system, and can be used also to take into account actuator dynamics in the protection. Another advantage is accurate tracking of the phase plane envelope, including portions for aggressive disturbance rejection. The technique is suitable for both control and command limiting, but is especially suitable for reference model controllers such as INDI or IBKS, Incremental Backstepping Controllers.

2 Envelope protections using phase plane

The following section explains the shaping of the response close to the boundary of the protected variable. As suggested by [11], [12], the method in this paper relies on shaping the response based on the phase plane of the protected variable. By phase plane is meant the relation between the protected variable and the derivative of the protected variable, i.e. specifying the limit of the derivative based on the current value of the variable, see Figure 1. By restricting the derivative of the protected variable it is possible to limit the speed at which the protected variable approaches the boundary based on the distance to it.

The concept is based on the calculation of the required state changes to achieve tracking of this phase plane limit, when the states approach the phase plane limit curve. With our proposal, this tracking is obtained by enforcing a limit on either the control/command signal, or the most upstream integrator in a reference model. In the case of angle of attack protection, this would employ a limit on the elevator deflection, or the pitch acceleration $\dot{q}$ integrator in a longitudinal reference model. In Figure 2 left is shown the concept of control limiting. P is the plant, i.e. the aircraft, C is the nominal controller and FEP is the envelope protection system. The reference r is from the pilot or the autopilot panel, u is the control signal, y is the output and x is the state feedback. The dotted line represents a logic switch, which changes the control signal to u_{fep}, when the control signal u exceeds the limit u_{fep}. In Figure 2 to the right, the limit is imposed on the command signal to the controller, all its features including disturbance rejection is maintained while in the control limiting

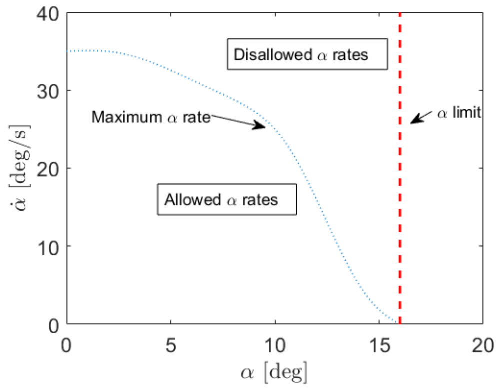

Fig. 1 Phase plane envelope of protected variable α

case, problems with integrator windup in the nominal controller has to be dealt with. In Figure 3 to the left, the protection is included in the reference model R, as a limit

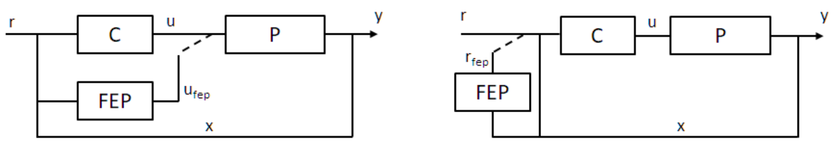

Fig. 2 *Left*: Control limiting flight envelope protection. *Right*: Command limiting flight envelope protection.

on the most upstream integrator. The FEP uses the states from the reference model x_{ref}, and the whole system runs in open loop. This can be beneficial for more high level protections such as energy or flight path angle, as the states of the reference model are noise free. In Figure 3 to the right, the FEP is used with the reference model in closed loop, i.e. real state measurements are fed back into the FEP. This has the advantage of including disturbance rejection. A combination of open and closed loop is also possible, to only feedback some measurements to the FEP.

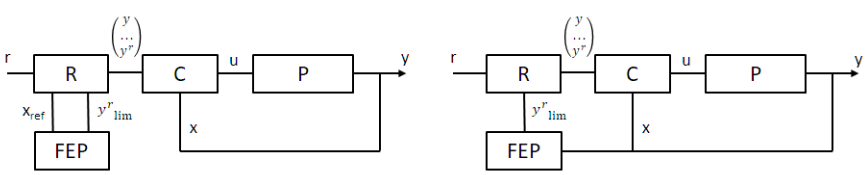

Fig. 3 *Left*: Open loop FEP in reference model *Right*: Closed loop FEP in reference model.

3 Output limiting with backstepping in the phase plane

This section introduces the developed concept for envelope protections. The approach is based on the idea of limiting and tracking within the phase plane. The derivation of the protection laws follows the back stepping procedure which is well known from e.g. [6] on a set of transformed coordinates. The reason for the introduction of the tilde and non-tilde notation in (1) and (4), is to better highlight the analogy with backstepping control, and for convenience in the derivations. The system considered is given in (1) in state space form. $\tilde{x} = [\tilde{x}_1, ...\tilde{x}_{n_x}]$ are the states that will be affected by the protection control and $\tilde{s} = [\tilde{s}_1, ...\tilde{s}_{n_s}]$ are slow dynamic states which are not controlled but influence the dynamics of the controlled states. The original states and functions of the system are denoted with a tilde. The transformed states and functions are without tilde. $\tilde{f}_n$, $\tilde{g}_n$ and $\tilde{d}_n$, $n = 1..n_x$ are possibly non-linear functions of both the controlled states and slow dynamic states, but are in this publication restricted to functions of the variables as shown in (1). This means that $\tilde{g}_n$ for the current scope are constants. $\tilde{d}$ and $\tilde{h}$ are the functions that describe the dynamics of the slow states as shown in (1). The formulation can be extended to the full unrestricted case, but requires several assumptions for the stability to be proven, which will be the subject of a later publication.

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dots \\ \dot{\tilde{x}}_{n_x} \\ \dot{\tilde{s}} \end{bmatrix} = \begin{bmatrix} \tilde{f}_1(\tilde{x}_1) + \tilde{g}_1\tilde{x}_2 + \tilde{d}_1(\tilde{s}) \\ \tilde{f}_2(\tilde{x}_1, \tilde{x}_2) + \tilde{g}_2\tilde{x}_3 + \tilde{d}_2(\tilde{s}) \\ \dots \\ \tilde{f}_{n_x}(\tilde{x}_1, ..., \tilde{x}_{n_x}) + \tilde{g}_{n_x}\tilde{u} + \tilde{d}_{n_x}(\tilde{s}) \\ \tilde{d}(\tilde{x}, \tilde{s}) + \tilde{h}\tilde{u} \end{bmatrix} \tag{1}$$

In case of a linear state space model, the structure can be written as:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{s}} \end{bmatrix} = \begin{bmatrix} A & P \\ Q & R \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{s} \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} 0_{1,n_x-1} & b_r \end{bmatrix}^T \\ B_s \end{bmatrix} \tilde{u} \tag{2}$$

Where $A \in \mathbb{R}^{n_x \times n_x}, P \in \mathbb{R}^{n_x \times n_s}, Q \in \mathbb{R}^{n_s \times n_x}, R \in \mathbb{R}^{n_s \times n_s}$ are matrices that describe the system dynamics, with n_x beeing the number of controlled states and n_s the number of slow dynamic states. b_r is a scalar that describes how the input affects the directly controlled state x_{n_x} and B_s is a vector that describes how the input affects the slow states $\tilde{s}$. In contrast to conventional backstepping as used for control design where the state $\tilde{x}_1$ is required to track a given trajectory y_r, in the context of envelope protections the state derivative $\dot{\tilde{x}}_1$ is required to track the given limiting trajectory $\dot{\tilde{x}}_1 \stackrel{!}{=} y_r = f\left(\tilde{x}_1, \tilde{x}_{1,lim}\right)$ in the phase plane. The system is transformed into a new set of states, $x = \dot{\tilde{x}}, s = \dot{\tilde{s}}$ and $u = \dot{\tilde{u}}$.

$$\begin{aligned} \begin{bmatrix} \ddot{\tilde{x}} \\ \ddot{\tilde{s}} \end{bmatrix} &= \begin{bmatrix} A & P \\ Q & R \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{s}} \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} 0_{1,n_x-1} & b_r \end{bmatrix}^T \\ B_s \end{bmatrix} \dot{\tilde{u}} \Rightarrow \\ \begin{bmatrix} \dot{x} \\ \dot{s} \end{bmatrix} &= \begin{bmatrix} A & P \\ Q & R \end{bmatrix} \begin{bmatrix} x \\ s \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} 0_{1,n_x-1} & b_r \end{bmatrix}^T \\ B_s \end{bmatrix} u \end{aligned} \tag{3}$$

Or if kept in the original notation, the similarities for use in the non-linear case can more readily be seen:

$$\begin{bmatrix} \dot{x}_1 \\ \dots \\ \dot{x}_{n_x-1} \\ x_{n_x} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} f_1(x_1) + g_1 x_2 + d_1(s) \\ \dots \\ f_{n_x-1}(x_1, \dots, x_{n_x-1}) + g_{n_x-1} x_{n_x} + d_{n_x-1}(s) \\ \tilde{f}_{n_x}(\tilde{x}_1, \dots, \tilde{x}_{n_x}) + \tilde{g}_{n_x} \tilde{u} + \tilde{d}_{n_x}(\tilde{s}) \\ d(x,s) + hu \end{bmatrix} \tag{4}$$

With $f_n(x_1, x_2, \dots x_n) = a_{n1}x_1 + \dots + a_{nn}x_n$, $g_n = a_{n(n+1)}$ and $d_n(s) = p_{n1}s_1 + \dots + p_{nn_s}s_{n_s}$ in case the system is linear on the form in (2). Now the problem has been transformed into the usual tracking problem, i.e. to make x_1 track y_r. The control law is synthesized using the concept of back stepping on the transformed system in (4). The error from the reference y_r and the state x_1 is formed in e_1.

$$e_1 = y_r - x_1 \tag{5}$$

The derivative of the error can be formulated as in (6), by substituting the dynamics for $\dot{x}_1$ as described by the system equations in (4).

$$\dot{e}_1 = \dot{y}_r - \dot{x}_1 = \dot{y}_r - f_1(x_1) - g_1 x_2 - d_1(s) \tag{6}$$

There exist several choices of Lyapunov functions, for example a Barrier one could be useful for this context, however, for the sake of simplicity, a quadratic Lyapunov function $V_1 = 1/2{e_1}^2$ is used in the further derivation. The derivative of V_1 with respect to time is given by (7).:

$$\dot{V}_1 = e_1\dot{e}_1 = e_1 \cdot [\dot{y}_r - f_1(x_1) - g_1 x_2 - d_1(s)] \overset{!}{\leq} -c_1 e_1^2 < 0 \tag{7}$$

The state x_2 is used as a virtual control input, to make the derivative of V_1 negative definite as required for Lyapunov stability. The desired $x_{2,d}$ which fulfils (7) is given in (8):

$$x_{2,d} = \hat{\tilde{x}}_{2,d} = \frac{1}{g_1} \left(\dot{y}_r - f_1(x_1) - d_1(s) + c_1 \left(y_r - x_1 \right) \right) \tag{8}$$

As $x_{2,d}$ cannot be directly controlled, it will differ from the actual x_2 by the error e_2. In the next design step, x_3 is used as a virtual input to achieve tracking $x_2 = x_{2,d}$ and the desired $x_{3,d}$ is calculated in (9) such that the extended Lyapunov function $V_2 = V_1 + \frac{1}{2} \cdot e_2^2$ has the negative definite time derivative $\dot{V}_2 = -c_1 e_1^2 - c_2 e_2^2$.

$$x_{3,d} = \left(\dot{x}_{2,d} - f_2(x_1, x_2) - d_2(s) + g_1 \cdot e_1 + c_2 e_2 \right) \frac{1}{g_2} \tag{9}$$

The relation in (9) depends on the time derivative of $x_{2,d}$ which is

$$\dot{x}_{2,d} = \left(\ddot{y}_r - \frac{\partial f_1}{\partial x_1}\dot{x}_1 - \frac{\partial d_1}{\partial s}\dot{s} + c_1 \overbrace{(\dot{y}_r - \dot{x}_1)}^{\dot{e}_1} \right) \frac{1}{g_1} \tag{10}$$

Regarding the derivatives involved in (10) for calculation of $\dot{x}_{2,d}$, the following assumptions and approximations can be made in the context of envelope protections for an aircraft. If the protection is included as open loop in the reference model, i.e. using the states of the reference model, all derivatives can be included without further considerations. If it is included with feedback from actual measurements, different possibilities exist for the calculation and inclusion of the time derivatives in (10). The derivative $\ddot{y}_r$ can be aproximated to only depend on $\tilde{x}_1$ as shown in (11) hence not on any derivatives of the states.

$$\begin{aligned} \dot{\tilde{x}}_{1,d} &= y_r = y_r(\tilde{x}_1) \\ \ddot{\tilde{x}}_{1,d} &= \dot{y}_r = \frac{\partial y_r}{\partial \tilde{x}_1}\dot{\tilde{x}}_{1,d} \\ \dddot{\tilde{x}}_{1,d} &= \ddot{y}_r = \frac{\partial y_r}{\partial \tilde{x}_1 \partial \tilde{x}_1}\dot{\tilde{x}}_{1,d}{}^2 + \frac{\partial y_r}{\partial \tilde{x}_1}\ddot{\tilde{x}}_{1,d} \end{aligned} \tag{11}$$

The second term in (10), $\frac{\partial f_1}{\partial x_1}\dot{x}_1$ is for flight envelope protections usually small and can be neglected, or included based on the model, and is kept here due to the stability consideration. The term $\frac{\partial d_1}{\partial s}\dot{s}$ can either be assumed small due to the time scale separation, i.e. $\dot{s} = 0$, or can be included only neglecting the influence of the derivative of the control inputs ($\dot{\tilde{u}} = u$) on the slow states, i.e. that $\frac{\partial d_1}{\partial s}hu \approx 0$. Hence, the influence of the fast states, x, and of s, on the slow states is kept. Only the effect of the control input rate $\dot{\tilde{u}}$ through the slow states is neglected, which in case of flight envelope protection is a reasonably assumption. For the last error term derivative, the largest contribution is the reference $\dot{y}_r$, and the time derivative $\dot{x}_1 = \ddot{\tilde{x}}_1$ needs to be estimated or filtered. If many back steps are involved, for example to include actuator dynamics, these desired virtual control input derivatives, $d^k/dt^k(x_{n,d})$ should be considered filtered using a command filtered back stepping method, see [5]. The presented procedure is continued for $n = 3, ..., n_x$ using (12).

$$x_{n,d} = \left(\dot{x}_{n-1,d} - f_{(n-1)}(x_1, ..., x_n) - d_{n-1}(s) + g_{n-2}e_{n-2} + c_{n-1}e_{n-1}\right) \frac{1}{g_{n-1}} \tag{12}$$

At this point, this value can be used to limit the most upstream integrator in a reference model. In many control applications with reference models as [1],[12] for example, in the case of angle of attack protection, this could be the $\dot{q}$ integration. In the case of limiting directly the control signal, the control signal to satisfy this virtual control can be immediately calculated from the original system (2):

$$\tilde{u} = (x_{n_x,d} - \tilde{f}_{n_x}(\tilde{x}_1, ..., \tilde{x}_{n_x}) - \tilde{d}_{n_x}(\tilde{s}))\frac{1}{\tilde{g}_{n_x}} \tag{13}$$

This last design step constitutes a slight difference to conventional backstepping where $\tilde{u}$ cannot be calculated directly and involves an additional back step.

This section showed that the back stepping approach can be used as a guidance for the construction of the protection control law with appropriate approximations and assumptions. In the following sections, this procedure denoted as OLB (Output Limiting with Backstepping) is used to synthesize protections for both angle of attack and flight path angle.

4 Angle of attack protection considering short period dynamics

This section demonstrates the just presented output limiting with backstepping (OLB) protection approach based on an simple example. The considered system is the linearised short-period dynamics of an aircraft. Based on this simple system an OLB alpha protection is derived and compared to an PPB (Phase Plane Based) alpha protection [11]. Simulation results reveal the benefits of OLB protections compared to PPB protections, which is first of all tracking in the phase plane. Another advantage of OLB is the possibility to apply piecewise polynomial phase plane limitations. A physically feasible exemplary piecewise polynomial is presented and applied to the OLB protection. The related simulations highlight the arising benefits. The most important ones are an increased disturbance rejection capability, improved performance (decreased conservatism) and stability as well as tracking in the phase plane.

4.1 System representation and derivation of OLB protection control law

The linearised short period dynamics of an aircraft are given as

$$\begin{aligned}\dot{\tilde{x}}_1 &= a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 \\ \dot{\tilde{x}}_2 &= a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + b\tilde{u}\end{aligned} \tag{14}$$

with the angle of attack $\tilde{x}_1 = \alpha$, the pitch rate $\tilde{x}_2 = q$ and the elevator deflection $\tilde{u} = \eta$. Transformation of the given system representation to the notation introduced in Section 3 results with $\tilde{s} = 0$ in

$$\begin{aligned}\dot{\tilde{x}}_1 &= \dot{x}_1 = f_1(x_1) + g_1 x_2 \\ \dot{\tilde{x}}_2 &= \tilde{f}_2(\tilde{x}_1, \tilde{x}_2) + \tilde{g}_2\tilde{u}\end{aligned} \tag{15}$$

with

$$\begin{aligned}f_1(x_1) &= a_{11}x_1 \\ g_1 &= a_{12} \\ \tilde{f}_2(\tilde{x}_1, \tilde{x}_2) &= a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 \\ \tilde{g}_2 &= b\end{aligned} \tag{16}$$

In case the protection is active, the time derivative of the angle of attack $x_1 = \dot{\tilde{x}}_1$, shall track the desired phase plane given as

$$y_r = f(\tilde{x}_1, \tilde{x}_{1,lim}) \tag{17}$$

Where $\tilde{x}_{1,lim}$ is the maximum allowable angle of attack α_{max}. The time derivative of the tracking error

$$e_1 = y_r - x_1 \tag{18}$$

is obtained as the following relation by inserting $\dot{x}_1$ from (15)

$$\dot{e}_1 = \dot{y}_r - \dot{x}_1 = \dot{y}_r - f_1(x_1) - g_1 x_2 \tag{19}$$

The time derivative of the Lyapunov control function candidate $V_1 = \frac{1}{2}{e_1}^2$ results in (20) by replacing $\dot{e}_1$ with the relation from (19).

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1 \left[\dot{y}_r - f_1(x_1) - g_1 x_2\right] \overset{!}{=} -c_1 {e_1}^2 \tag{20}$$

For $c_1 > 0$ it holds that $\dot{V}_1 < 0$. Solving

$$\dot{y}_r - f_1(x_1) - g_1 x_2 \overset{!}{=} -c_1 e_1 \tag{21}$$

which is obtained from (20), for x_2 leads to the virtual control law

$$x_{2,d} = \dot{\tilde{x}}_{2,d} = \frac{1}{g_1}\left(\dot{y}_r - f_1(x_1) + c_1 e_1\right) \tag{22}$$

Thus, the system is globally asymptotically stable in the sense of Lyapunov if $x_2 = x_{2,d}$. From the $\tilde{x}_2$ subsystem in (15) we can directly calculate the control law $\tilde{u}$ accordiung to (23) such that x_2 tracks $x_{2,d}$.

$$\tilde{u} = \frac{1}{\tilde{g}_2}\left(x_{2,d} - \tilde{f}_2(\tilde{x}_1, \tilde{x}_2)\right) \tag{23}$$

4.2 Linear phase plane

First of all the OLB protection derived in Section 4.1 is compared to the PPB protection scheme in [11]. Since the PPB scheme only incorporates linear phase planes of the form

$$y_r = K_P(\tilde{x}_{1,\text{lim}} - \tilde{x}_1) \tag{24}$$

the same phase plane is applied to the OLB protection law. At this point it shall be noted that for a linear phase plane according to (24) and a linear system according to (15), the control law $\tilde{u}$ in (23) constitutes a linear state feedback law: Insertion of (22) into (23) and substitution of the relation (18) for e_1 yields

$$\tilde{u} = \frac{1}{\tilde{g}_2}\left(\frac{1}{g_1}\left(\dot{y}_r - f_1(x_1) + c_1(y_r - x_1)\right) - \tilde{f}_2(\tilde{x}_1, \tilde{x}_2)\right) \tag{25}$$

Replacing $x_1 = \dot{\tilde{x}}_1$ by the relation given in (14) as well as f_1 and $\tilde{f}_2$ given in (16) results in

$$\begin{aligned} \tilde{u} &= \tfrac{1}{\tilde{g}_2}\left(\tfrac{1}{g_1}\left(\dot{y}_r - a_{11}x_1 + c_1(y_r - x_1)\right) - a_{21}\tilde{x}_1 - a_{22}\tilde{x}_2\right) = \\ &\tfrac{1}{\tilde{g}_2}\left(\tfrac{1}{g_1}\left(\dot{y}_r - a_{11}(a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2) + c_1(y_r - (a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2))\right) - a_{21}\tilde{x}_1 - a_{22}\tilde{x}_2\right) \end{aligned} \tag{26}$$

Finally the state feedback structure is obtained by inserting y_r and its derivative which is (incorporating that $\dot{\tilde{x}}_1$ shall track the desired reference trajectory y_r)

$$\dot{y}_r = -K_P\dot{\tilde{x}}_1 = -K_P y_r = -K_P^2(\tilde{x}_{1,lim} - \tilde{x}_1). \tag{27}$$

into (26) as

$$\tilde{u} = -L\begin{bmatrix}\tilde{x}_1 \\ \tilde{x}_2\end{bmatrix} + h\tilde{x}_{1,lim} \tag{28}$$

In (28) L and h constitute the constant feedback gain matrix and feedforward gain, respectively. These are

$$\begin{aligned} L &= \begin{bmatrix} l_1 & l_2 \end{bmatrix} \\ l_1 &= \tfrac{1}{\tilde{g}_2}\left(\tfrac{1}{g_1}(a_{11}{}^2 + c_1 a_{11} - K_P^2 + c_1 K_P) + a_{21}\right) \\ l_2 &= \tfrac{1}{\tilde{g}_2}\left(\tfrac{1}{g_1}(a_{11}a_{12} + c_1 a_{12}) + a_{22}\right) \\ h &= \tfrac{1}{\tilde{g}_2}\tfrac{1}{g_1}(c_1 K_P - K_P^2) \end{aligned} \tag{29}$$

Application of this feedback law to (14) which can be formulated as the statespace representation $\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$, yields the closed- loop dynamics

$$\dot{\tilde{x}} = (A - BL)\tilde{x} + Bh\tilde{x}_{1,lim}. \tag{30}$$

The eigenvalues of the closed-loop dynamics $(A - BL)$ can be easily computed to proof stability of the closed-loop.

If $x_2 = x_{2,d}$ holds then $\tilde{x}_2 = \frac{1}{a_{12}}(y_r - a_{11}\tilde{x}_1 + \int(c_1 e_1)dt)$. Inserting this relation into $x_1 = a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2$ yields

$$\begin{aligned} x_1 = \dot{\tilde{x}}_1 &= y_r + \int(c_1(y_r - \dot{\tilde{x}}_1))dt \\ &= y_r + \int(c_1 y_r)dt - c_1\tilde{x}_1 \end{aligned} \tag{31}$$

Substitution of y_r from (24) finally results in

$$\dot{\tilde{x}}_1 = -(K_P + c_1)\tilde{x}_1 - \int(c_1 K_P \tilde{x}_1)dt + K_P\tilde{x}_{1,lim} + \int(c_1 K_P \tilde{x}_{1,lim})dt \tag{32}$$

The corresponding Laplace transform is

$$s\tilde{X}_1(s) = -(K_P + c_1)\tilde{X}_1(s) - \frac{1}{s}c_1 K_P\tilde{X}_1(s) + K_P\tilde{X}_{1,lim}(s) + \frac{1}{s}c_1 K_P\tilde{X}_{1,lim}(s) \tag{33}$$

Solving for $\tilde{X}_1(s)$ leads to the transfer behaviour

$$\tilde{X}_1(s) = \frac{K_P s + c_1 K_P}{s^2 + (K_P + c_1)s + c_1 K_P}\tilde{X}_{1,lim}(s) \tag{34}$$

This is the resulting closed loop behaviour of the protection dynamics. If we let c_1 go to infinity, we see that the corresponding dynamics is the expected 1st order behavior of $\tilde{x}_1$.

$$\tilde{X}_1(s) = \frac{\frac{K_P}{c_1}s + K_P}{\frac{s^2}{c_1} + \frac{K_P}{c_1}s + s + K_P}X_{1,lim}(s) \rightarrow \frac{K_P}{s + K_P}\tilde{X}_{1,lim}(s), c_1 \rightarrow \infty \tag{35}$$

This shows that the protected dynamics follow the desired dynamics defined by the protection function y_r in the limit, hence the c_1 gain defines the aggressiveness with which the protected dynamics follow the desired response dynamics. This dedicated behaviour is incorporated into the the polynomial shaping introduced in Section 4.3. As will be revealed later the polynomial is shaped in such a way that close to the limit the response corresponds to the linear one above. A linear behaviour close to the limit as described above is advantegous as the timeconstant $\tau_p = 1/K_P$ defines the predicted time to limit violation. This predicted time to limit violation describes the time from the current state $\tilde{x}_1$ until its limit $\tilde{x}_{1,lim}$ is reached, given that the derivative $\dot{\tilde{x}}_1$ is maintained, as depicted in Figure 4. The comparison of PPB protection and the proposed OLB protection in the phase plane shown in Figure 4, reveals that OLB is capabable of tracking the limiting function while PPB is not. This is important in terms of showing compliance with the required behaviour defined by the phase plane envelope.

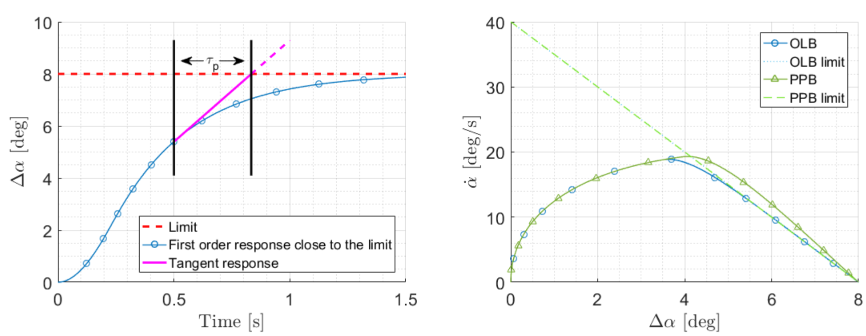

Fig. 4 *Left*: Visualization of predicted time to limit *Right*: Comparison of phase plane tracking of OLB and PPB angle of attack protection

4.3 Piecewise polynomial phase plane

This section introduces the physically feasible polynomial phase plane used in the angle of attack protection derived in Section 4.1 and highlights the arising advantages which are increased performance and improved disturbance rejection. The piecewise polynomial consists of three polynomials. At the transition point the first and second polynomial are equal up to the derivatives required for the control law developed in Section 4.1. The desired phase plane is shaped as

$$y_r = \begin{cases} a_0 + a_1\tilde{x}_1 + a_2\tilde{x}_1^2 + a_3\tilde{x}_1^3 + a_4\tilde{x}_1^4 + a_5\tilde{x}_1^5 + a_6\tilde{x}_1^6 & \text{for } 0 < \tilde{x}_1 < \tilde{x}_{1,\text{activate}} \\ b_0 + b_1\tilde{x}_1 + b_2\tilde{x}_1^2 + b_3\tilde{x}_1^3 + b_4\tilde{x}_1^4 + b_5\tilde{x}_1^5 + b_6\tilde{x}_1^6 & \text{for } \tilde{x}_{1,\text{activate}} < \tilde{x}_1 < \tilde{x}_{1,\text{lim}} \\ K_r(\tilde{x}_{1,\text{lim}} - \tilde{x}_1) & \text{for } \tilde{x}_{1,\text{lim}} < \tilde{x}_1 \end{cases} \tag{36}$$

Figure 5 depicts the resulting phase plane as well as the linear phase plane used in Section 4.2.

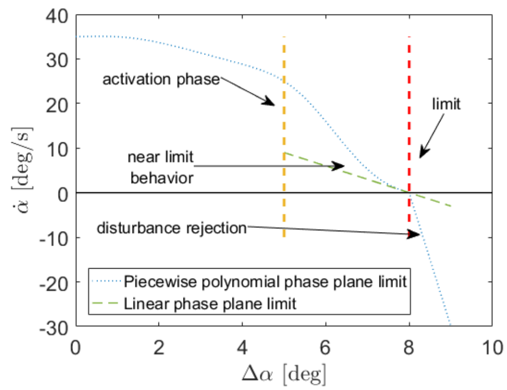

Fig. 5 Piecewise polynomial phase plane and linear phase plane

Figure 6 compares the phase plane response and time domain response of the OLB protection with the linear phase plane (24) and the OLB protection with the polynomial phase plane (36). Application of the polynomial phase plane limit leads to an improved performance as can be seen in both the time domain and phase plane. The nominal, unprotected response is tracked far longer. Hence the protection with the linear phase plane limit is more conservative. Close to the limit the polynomial phase plane corresponds to the linear phase plane in order to approach the limit with a linear behaviour specified by the time to exceedance τ_p.

Due to the increased slope of the polynomial phase plane compared to the linear phase plane above the limit $\tilde{x}_{1,lim}$, the disturbance rejection is increased. The reason is that the slope defines the degree of aggressiveness in pushing back the state $\tilde{x}_1$ to its limit once it is violated due to a gust for example. Figure 7 compares the disturbance rejection characteristics of the OLB protection with linear phase plane

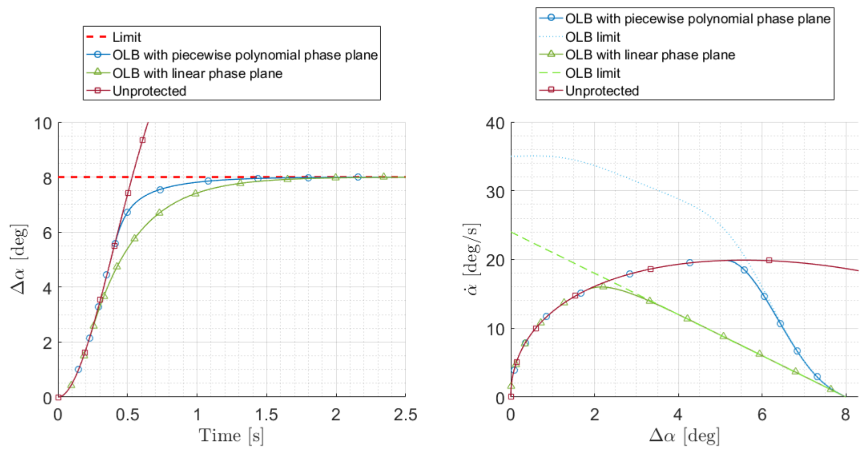

Fig. 6 *Left*: Comparison of time domain response of unprotected Angle of attack and protected with linear and polynomial phase plane *Right*: Comparison of phase plane of unprotected and OLB with linear and polynomial phase plane limit

and of the OLB protection with the introduced piecewise polynomial phase plane. The applied disturbance is a $1-cosine$ gust with maximum certification amplitude according to [9]. The applied disturbance is depicted in Figure 7. As expected, the increased slope above $\tilde{x}_{1,lim}$ of the polynomial phase plane leads to an considerably improved disturbance rejection.

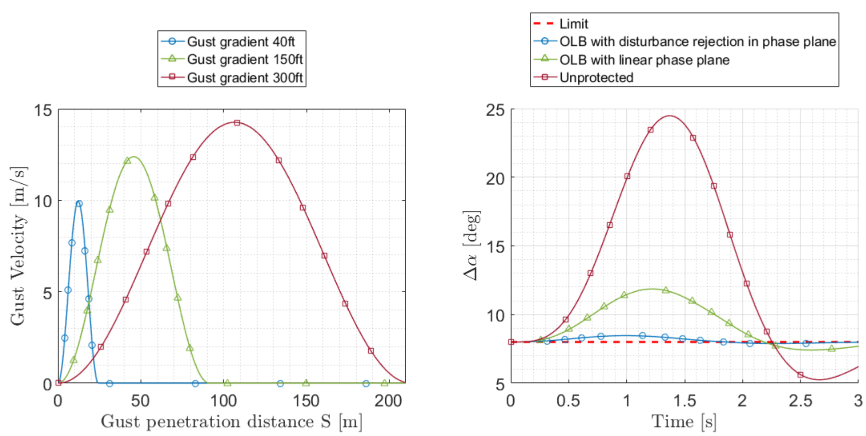

Fig. 7 *Left*: Applied $1-cosine$ gust *Right*: Comparison of unprotected response and of OLB response with linear and polynomial phase plane limit subjected to disturbance

5 Flight path angle protection considering coupled linearised longitudinal dynamics

In order to introduce the concept and to highlight its most important characteristics it was demonstrated in the previous section for an alpha protection based on the linearised short period dynamics of an aircraft considering only pitch rate and angle of attack as states of the system. States as velocity V and flight path angle γ could have been included as well, but were neglected for simplicity in the explanation. In the following example two additional states of the longitudinal motion are taken into account, velocity and flight path angle. To demonstrate the full concept, flight path angle is now to be limited, which in contrast to the angle of attack protection, requires an additional backstepping loop. The velocity V is considered as a slow state according to Section 3. Simulation results reveal that the developed concept successfully protects the desired variable, also in the case where multiple back steps are required as well as its capability to handle the additional influences on the protected dynamics arising from the slow state dynamics.

5.1 System representation and protection control law

The considered system are the linearised dynamics of the longitudinal motion of an aircraft, given as the state space representation in (37).

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \dot{\tilde{s}}_1 \end{bmatrix} = \begin{bmatrix} \dot{\gamma} \\ \dot{\alpha} \\ \dot{q} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} A & P \\ Q & R \end{bmatrix} \begin{bmatrix} \gamma \\ \alpha \\ q \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_r \\ b_s \end{bmatrix} \tilde{u} \tag{37}$$

with $A \in \mathbb{R}^{3\times3}, P \in \mathbb{R}^{3\times1}, Q \in \mathbb{R}^{1\times3}, R \in \mathbb{R}^{1\times1}$, the flight path angle $\gamma = \tilde{x}_1$, angle of attack $\alpha = \tilde{x}_2$ and pitch rate $q = \tilde{x}_3$ and the slow dynamic state, velocity $V = \tilde{s}_1$. The input $\tilde{u} = \eta$ is the elevator deflection. This system can be represented in the notation introduced in Section 3 as (38).

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \dot{\tilde{s}}_1 \end{bmatrix} = \begin{bmatrix} \tilde{f}_1(\tilde{x}_1) + \tilde{g}_1\tilde{x}_2 + \tilde{d}_1(\tilde{s}_1) \\ \tilde{f}_2(\tilde{x}_1,\tilde{x}_2) + \tilde{g}_2\tilde{x}_3 + \tilde{d}_2(\tilde{s}_1) \\ \tilde{f}_3(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3) + \tilde{g}_3\tilde{u} + \tilde{d}_3(\tilde{s}_1) \\ \tilde{d}(\tilde{x},\tilde{s}_1) + \tilde{h}\tilde{u} \end{bmatrix} \tag{38}$$

As described in Section 3 the system is further transformed into (39) by differentiation and the change of coordinates $\dot{\tilde{x}}_i = x_i, \dot{\tilde{s}}_i = s_i$.

$$\begin{bmatrix} \ddot{\tilde{x}}_1 \\ \ddot{\tilde{x}}_2 \\ \ddot{\tilde{x}}_3 \\ \ddot{\tilde{s}}_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{s}_1 \end{bmatrix} = \begin{bmatrix} f_1(x_1) + g_1 x_2 + d_1(s_1) \\ f_2(x_1, x_2) + g_2 x_3 + d_2(s_1) \\ f_3(x_1, x_2, x_3) + g_3 u + d_3(s_1) \\ d(x, s_1) + hu \end{bmatrix} = \begin{bmatrix} A & P \\ Q & R \end{bmatrix} \begin{bmatrix} \dot{\gamma} \\ \dot{\alpha} \\ \dot{q} \\ \dot{V} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_r \\ b_s \end{bmatrix} \dot{\tilde{u}} \quad (39)$$

First of all $x_{2,d} = \dot{\alpha}_d$ is calculated according to (8). Therefore, $x_1 = \dot{\gamma}$ is estimated using the relation $\dot{\gamma} = \tilde{f}_1(\tilde{x}_1) + \tilde{g}_1\tilde{x}_2 + \tilde{d}_1(\tilde{s}_1)$ given by (38) with the measured flight path angle $\tilde{x}_1$, angle of attack $\tilde{x}_2$ and velocity $\tilde{s}_1$. Furthermore $d_1(s_1)$ is calculated (with the estimate $s_1 = \dot{V}$ using (37)) as

$$d_1(s_1) = p_{11}\dot{V} = p_{11}(q_{11}\gamma + q_{12}\alpha + q_{13}q + r_{11}V + b_s\tilde{u}) \quad (40)$$

with $\mathrm{p}_{11} b_s \approx 0$, i.e. the elevators direct influence on the flight path angle dynamics through the velocity dynamics is neglected. Here p_{ij}, q_{ij} and r_{ij} refers to the entries of matrices P, Q and R respectively. In the next step $x_{3,d} = \dot{q}_d$ according to (9) is calculated. Therefore, the time derivatives $\dot{x}_1 = \ddot{\gamma}$ and $\dot{s}_1 = \ddot{V}$ are required to obtain $\dot{x}_{2,d}$, calculated as in equation (10). $\dot{x}_1$ is estimated as $\dot{x}_1 = f_1(x_1) + g_1 x_2 + d_1(s_1)$ according to (39). Thereby $x_1 = \dot{\gamma}$ and $d_1(s_1)$ from the previous step can be used. The estimate $x_2 = \dot{\alpha}$ is obtained over $\dot{\alpha} = \tilde{f}_2(\tilde{x}_1, \tilde{x}_2) + \tilde{g}_2\tilde{x}_3 + \tilde{d}_2(\tilde{s}_1)$ (38), where additionally the measured pitch rate $\tilde{x}_3$ is used. The time derivative $\dot{s}$ follows $\dot{s} = q_{11}\dot{\gamma} + q_{12}\dot{\alpha} + q_{13}\dot{q} + b_s\dot{\tilde{u}}$. In (10) we insert the relation $\frac{\partial d_1}{\partial s_1}\dot{s}_1 = p_{11}\dot{s}_1$, incorporating $\mathrm{p}_{11} b_s \approx 0$ and $\mathrm{p}_{11} q_{13} b_r \approx 0$ as $q_{13} \approx 0$, what is reasonable as the phugoid dynamics are not influenced by the pitch rate dynamics. The procedure to verify stability reveals the required assumptions that need to be satisfied for the system. These assumptions vary between different protections, e.g. in the γ protection, b_s and q_{13} needed to be zero, which is not necessarily the case for other protections. Finally $\tilde{u}$ is calculated as presented in (13).

$$\tilde{u} = (x_{3,d} - \tilde{f}_3(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) - \tilde{d}_3(\tilde{s}_1))\frac{1}{\tilde{g}_3}. \quad (41)$$

The control signal $\tilde{u}$ can either be used as control limiting for the elevator η, or the variable $x_{3,d} = \dot{q}_d$ can be used as a limiter in the reference model such that the maximum pitch acceleration is limited by the desired $\dot{q}_d$ wrt. tracking the phase plane limit.

5.2 *Phase plane tracking, uncertainties and noise*

The phase plane envelope is again constructed as a piecewise polynomial function according to (36). The nominal phase plane tracking is shown in Figure 8 to the right, and the corresponding time domain response is shown to the left. If implemented as open loop in a reference model, the tuning allows a more free choice of tuning parameters c_1 and c_2 as the effect of noise does not propagate in the back stepping. If on the other hand, the loop is closed with noisy measurements, the gain tuning has to be somewhat relaxed, and the control signal has to be smoothed. In

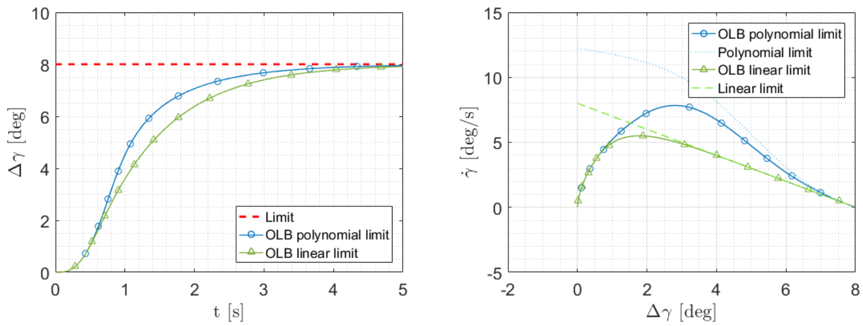

Fig. 8 *Left*: Time domain response of gamma protection with linear and polynomial phase plane. *Right*: Phase plane response of gamma protection with linear and polynomial phase plane.

Figure 9 the response is shown with discrete white noise added on the γ measurement with $\sigma = 0.2deg$, on α measurement with $\sigma = 0.5deg$ and on the pitch rate measurement, q, with $\sigma = 0.1deg/s$ at a $100Hz$ sampling rate. The control signal $\tilde{u}$ has been smoothed using a first order filter. Figure 9 also illustrates the response in case of uncertainties, where 10% is added to all coefficients in the model. Both cases demonstrate that the proposed protection concept is capabable to achieve tracking of the phase plane limitations even in case of realistic levels of uncertainties and noise. The gains have been chosen to $c_1 = 10$ and $c_2 = 9$, such that from steady state flight with maximum command towards the limit, the envelope protection command does not saturate in the control signal. The tuning of the phase plane limits and the gains are subject for another publication, but should be chosen incorporating aircraft performance, actuator constraints/rate constraints, disturbance analysis, uncertainties and measurement noise and should be scheduled over the entire flight envelope.

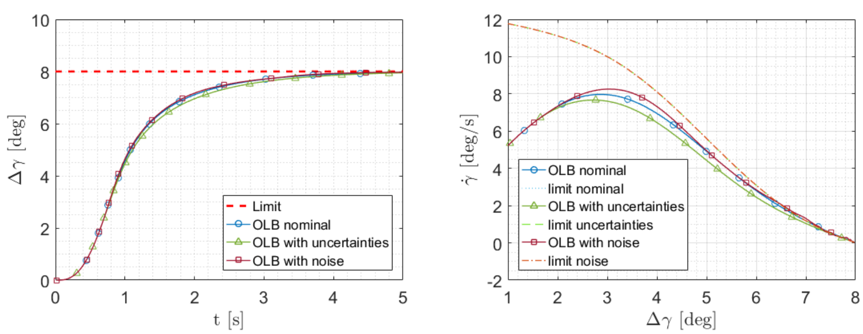

Fig. 9 *Left*: Time domain response of gamma protection with uncertainties and noise. *Right*: Phase plane response of gamma protection with uncertainties and noise.

6 Conclusion

In this paper, a strategy for phase plane based flight envelope protection using a back stepping type approach was evaluated with proven stability in the protection domain and verified through simulation analysis. The method is able to successfully track sufficiently smooth phase plane limitations, also in case of realistic levels of uncertainty and noise. The approach is capable of taking into account the transient response behaviour, and can incorporate aggressive disturbance rejection above the limit of a protected variable. By utilizing piecewise polynomial limits in the phase plane, a less conservative protection is established, enhancing aircraft performance around the limit of protected variables. The approach has a well defined and intuitive response behaviour close to the limit, in which the distance from from the limit is defined with a constant time to violation. Further research will go in the direction of non-linear models for angle of attack protection, together with phase plane envelope adaptation based on flight conditions and actuator failures.

Acknowledgements This research is co-funded by the European Union in the scope of INCEPTION project, which has received funding from the EU's Horizon2020 Research and Innovation Programme under grant agreement No. 723515.

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