Dynamic analysis of tire consumption in aircraft anti-skid braking

Luca d'Avico, Mara Tanelli and Sergio M. Savaresi

Abstract Tire consumption is a crucial element in determining the maintenance costs of aircraft. Clearly, it has a strong link with anti-skid controllers. In fact, in aircraft braking, nearly all braking maneuvers activate the anti-skid controller, which remains in use for long time intervals. This is not true in ground vehicles, as anti-skid control is usually active for a small part of braking maneuvers and in general for a short time only. Thus, tire consumption in the automotive context is usually related to the mileage covered, and it is studied under constant speed assumptions. In this work, we extend existing tire consumption models to consider the braking dynamics explicitly, and we show that, using appropriate anti-skid control approaches, tire wear can be directly linked to the controller parameters, thus offering a way to limit tire consumption, and hence maintenance costs, by properly tuning the controller itself.

1 Introduction

Anti-skid control systems have been installed on aircraft for several decades, being the precursors of those that were later tested and developed for ground vehicles, [13]. In the current industrial practice, anti-skid systems are mainly designed by braking systems suppliers as *black-boxes* that come with the braking actuation of the landing gear system, and only need to be connected with the wheel speed sensor. Each landing gear has a dedicated anti-skid system which might in some cases share some information with the other local anti-skids (for example to design high-level logics that are based on the wheel speed difference in different landing gears), but it does not in general communicate with other parts of the aircraft, [8].

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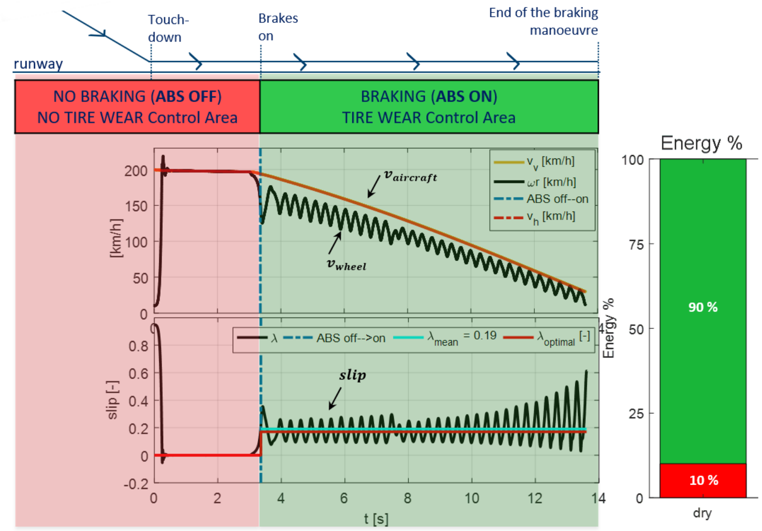


Fig. 1 General description of the impact of tire wear phenomenon in aircraft braking.

In aircraft, the anti-skid is in general activated on most braking maneuvers, both in the case of landing and rejected take off (RTO). Moreover, once activated, the anti-skid stays active for the most part of the braking itself, until the aircraft reaches a nearly standstill condition. This means that the wheel skid induced by the controller has a long time over which it acts continuously on the wheels, thus inducing a significant wear and consumption of the tires. In fact, the cost of changing tires that reach their end of life is one of the most relevant in aircraft maintenance.

To fix the scope of the considered problem, let us refer to Figure 1, which shows on the top the different phases of an aircraft landing, and, on the bottom, a possible corresponding time history of speed and skid, assuming that an anti-skid controller is in place. Of course, being interested in investigating the impact of the anti-skid control on tire wear, one wants first to ascertain that the amount of energy that is actually involved in the controlled braking phase is indeed much larger that that spent in the touch-down time interval. The right part of Figure 1 shows that the energy share of the two phases is of approximately 10% for the touch down (when the still wheels are abruptly put in contact with the ground and experience a very strong skid once they feel the very large forward speed induced by the aircraft inertia at the beginning of the wheel-on-ground phase) and 90% for the braking phase itself. This allows us to claim that, in case we can relate the anti-skid tuning with the resulting tire-wear, then we have a means to significantly vary the tire consumption by acting on the controller itself. Of course, to be of interest, such a reduction in consumption must not result in an excessive degradation of the braking performance.

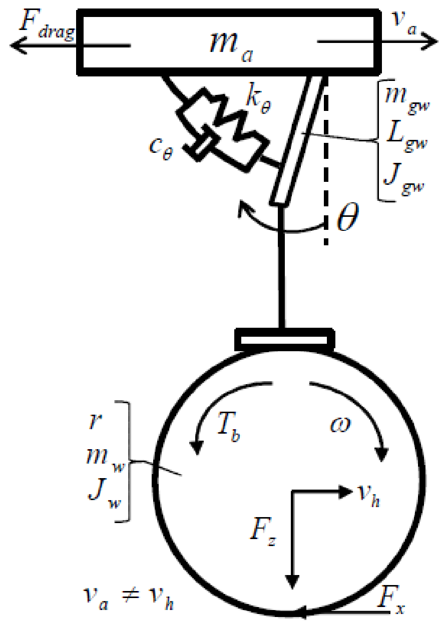


Fig. 2 Control-oriented dynamic model of an aircraft landing gear.

This work focuses on such an analysis, providing a dynamic model of the tire consumption mechanism that can be used during braking, and then studies the interplay between anti-skid control and tire wear. Specifically, two different control algorithms are considered. One is in line with the current industrial practice, being designed using the only measurement provided by the wheel speed sensor, and yielding a final cyclic skid behavior such as that shown in the bottom part of Figure 1, see [5]. The second one is a more advanced anti-skid control approach, which uses a measure or an estimation of the aircraft velocity and designs a genuine skid regulation system, see [4]. Our analysis will show that the main driver in moving from the traditional to the new design approach is to be found in the much increased capability of acting on the tire wear phenomenon.

The obtained results lead to the idea that tire consumption can actually be controlled *via* anti-skid control design. This concept, and its further elaboration, have been recently protected with a patent application, [14].

The rest of the paper is structured as follows: Section 2 introduces the dynamic model of the landing gear, which is used as a basis for anti-skid controller design. Further, Section 3 presents the dynamic tire consumption model proposed in this work, and Section 4 introduces the anti-skid control algorithms considered in this work. Finally, Section 5 performs the combined analysis of anti-skid control and tire consumption, showing the direct link between the two.

2 Braking dynamics model

For modelling the aircraft dynamics during braking for anti-skid design, it is worth focusing on the dynamics of the landing gear, assuming that the mass insisting on such element is that of approximately half aircraft. Such condensed modelling is motivated by the fact that industrial anti-skid control systems are embedded in the single braking system, each installed on one of the two landing gears, see *e.g.*, [6, 16]. Such model is the aeronautic counterpart of the so-called single-corner model used for braking control of vehicles, [15]. To obtain a complete model of the landing gear for anti-skid control problems, the wheel dynamics must be described; to this end, the wheel slip ratio is defined as

$$\lambda = \frac{v_h - \omega r}{v_h}, \tag{1}$$

where v_h is the wheel hub speed, r is the wheel radius and ω is the angular wheel speed. The hub speed is defined as

$$v_h = v_a - \dot{\theta} L_{gw}, \tag{2}$$

where v_a is the aircraft longitudinal speed, $\dot{\theta}$ is gear walk angular speed and L_{gw} is the length of the link between the chassis and the wheel. The tire-runway interaction is defined by the longitudinal force F_x as a function of the vertical load and the wheel slip defined as

$$F_x = F_z \mu(\lambda). \tag{3}$$

In this formulation, $\mu(\lambda)$ is the longitudinal friction coefficient, which is a function of the wheel slip and describes the available friction depending on the runway types of surfaces. Various empirical analytical expressions of $\mu(\lambda)$ were proposed in the literature. A widely-used expression (see *e.g.*, [7, 15]) is the so-called Burckhardt model

$$\mu(\lambda) = \vartheta_1(1 - e^{-\lambda\vartheta_2}) - \lambda\vartheta_3, \tag{4}$$

where the description of the road surface is given by ϑ_i, $i = 1,2,3$. Different values of these parameters allow to model different tire-road friction conditions; the curve $\mu(\lambda)$ in four various conditions is displayed in Fig. 3.

The main difference between the control-oriented description of the landing gear and the automotive single-corner model is the presence of the so-called *gear walk* phenomenon. Such a phenomenon can be describes as an oscillatory motion of the landing gear in the longitudinal direction taking place about a normally static vertical center line. This motion is due to the interaction between tire and runway modulated by the vertical load which deflects the landing gear. Such an oscillation may of course interact with the anti-skid closed-loop behavior, and must be considered in the design of anti-skid systems for aircraft, see *e.g.*, [8, 16]. The control-oriented

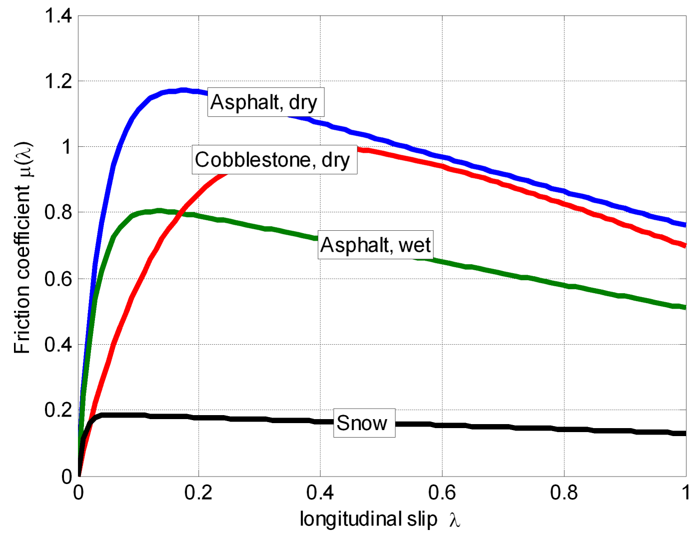


Fig. 3 Burckhardt model of the friction coefficient $\mu(\lambda)$.

view of the gear-walk phenomenon in a landing gear can be compactly described as a rotational spring-damper, as shown in Fig. 2. Such a phenomenon can be described as an oscillatory motion of the landing gear in the longitudinal direction, taking place around a static vertical center line. The main reason for this behavior is the tire-runway friction condition modulated by the vertical load. It must be taken into account in the anti-skid design since it can be negatively coupled with the anti-skid closed-loop control action (see *e.g.* [16]).

In the modeling of the landing gear, a constant wheel radius and static vertical load are considered. Both assumptions come from the single-corner approach, the starting point for this paper, which does not take into account the suspension dynamics. In the automotive field, neglecting the suspension dynamics during the braking control system design, results in only a retuning of the controller parameters; this takes into account the load transfer dynamics and it generally produces different controller parameters for the front and rear wheels. This approach relies upon the fact that wheel dynamics are sufficiently faster than chassis and suspension excitation frequencies. To our best knowledge, the combination of the gear-walk, suspensions and longitudinal motion has not been extensively studied in the aeronautic field from the anti-skid design point of view yet. In principle, the gear-walk motion can introduce a strong coupling between the longitudinal and vertical dynamics; this subject is currently under study.

The non-linear model of the aircraft landing gear, which is schematically represented in Fig. 2, can be described as

$$(m_a + m_w + m_{gw})\dot{v}_a - J_\theta \ddot{\theta} = -F_x - F_{drag} \quad (5)$$
$$J_{\ddot{\theta}} \ddot{\theta} - J_\theta \dot{v}_a + c_\theta \dot{\theta} + k_\theta \theta = L_{gw} F_x \quad (6)$$
$$J_w \dot{\omega} = r F_x - T_b, \quad (7)$$

where
$J_\theta = \left(\frac{L_{gw} m_{gw}}{2} + L_{gw} m_w\right)$, $J_{\ddot{\theta}} = \left(J_{gw+} \frac{L_{gw}^2 m_{gw}}{4} + L_{gw}^2 m_w\right)$ and $F_{drag} = \alpha_d v_a^2$ is the drag force. The parameter α_d is tuned based on experimental data that are not reported here for sake of brevity.

Using experimental data, see [3], the actuator dynamics were modeled with the second order transfer function

$$A(s) = \frac{\omega_n^2}{s^2 + 2\xi_n \omega_n s + \omega_n^2}, \quad (8)$$

with $\xi_n \approx 0.7$ and $f_n = \frac{\omega_n}{2\pi} \approx 15$Hz, which is used in the simulation setting.

3 Tire Wear Modelling

The tire wear phenomenon is due to several mechanisms ranging from mechanical delamination to rubber oxidation [2,10]. It is strongly non-linear, and it is influenced by numerous variables, such as contact geometry, the presence of contaminants between contact surfaces, tire temperature and pressure, to name the most important. Furthermore, see also [1,9], the most relevant dynamic factors are the longitudinal vehicle speed, the longitudinal slip and the tire side-slip angle, *i.e.*, the angle between the tire longitudinal axis and the direction of the tire longitudinal speed. As in this work the focus is on longitudinal braking maneuver, as those performed during landings and RTOs, in what follows the tire side-slip angle can be considered null, as tire longitudinal axis and tire speed are aligned.

In the automotive literature, where tire consumption has been mainly studied, the phenomenon has been analyzed considering constant-speed driving, as in cars, and ground vehicles in general, the main driver of tire consumption is usage. In fact, strong braking maneuvers happen quite rarely, and are also rather short. In aircraft, instead, the contact between tires and road is rather limited, and braking maneuvers are always performed with maximal braking force request, thus requesting the anti-skid controller action. Thus, the model available in the literature must be extended to encompass the effect of the braking dynamics. To do this, consider that, from a physical viewpoint, tire wear is generated by the difference between the longitudinal frictional power $F_x v$ and the braking power $T_b \omega$; this second term can in turn be obtained from the rotational wheel dynamics in (7). Therefore, a comprehensive model of the tire-wear power (TW) can be defined as

$$TW = F_x(v_h - \omega r) + J_\omega \omega |\dot{\omega}| = F_x v_h \lambda + J_\omega \omega |\dot{\omega}|. \quad (9)$$

Note that the second summand represents the contributions to the tire wear due to the typical wheel speed oscillations introduced by the ABS control algorithm, which, see *e.g.*, [5, 11], and it is typically disregarded in the automotive literature, [1]) for the reasons explained above.

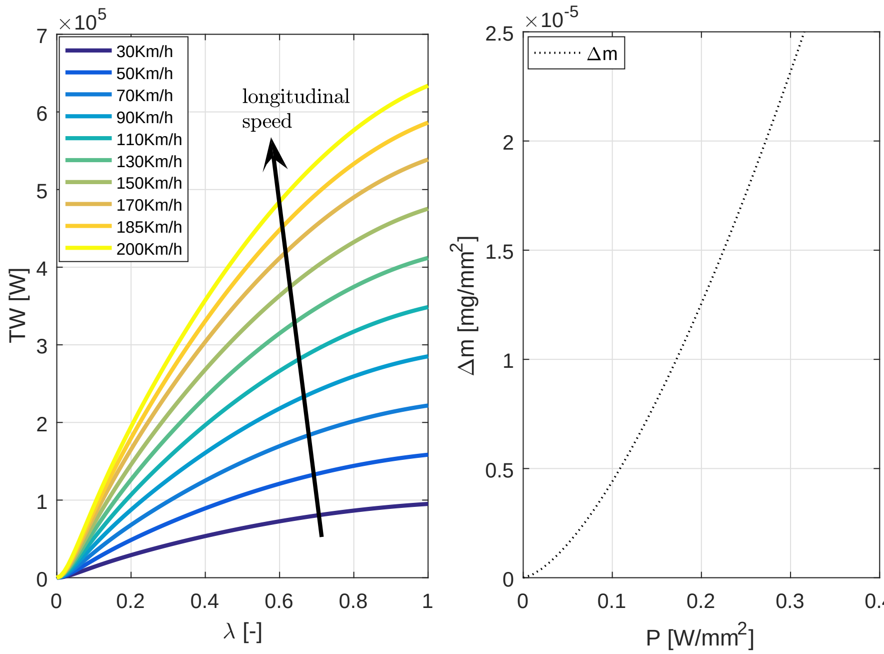


Fig. 4 Tire wear phenomenon. (a): Tire-wear power as a function of the wheel slip for different values of the longitudinal speed; (b) Tire mass-loss as a function of the tire-wear power.

In Fig. 4a, a steady-state sensitivity analysis of (9) is shown. By inspecting the left figure, one has confirmation of the physical interpretation of the tire wear phenomenon described: as wheel slip and longitudinal speed increase, a larger tire wear power is generated, thus increasing the overall tire consumption.

Furthermore, according to [2, 9], the mass loss per unit contact area can also be computed according to

$$\Delta m = f_1 P^{f_2}, \tag{10}$$

where $P = TW/a_{cont}$ and a_{cont} is the contact area between tire and road. In Fig. 4b, the results obtained using Equation (10) is reported.

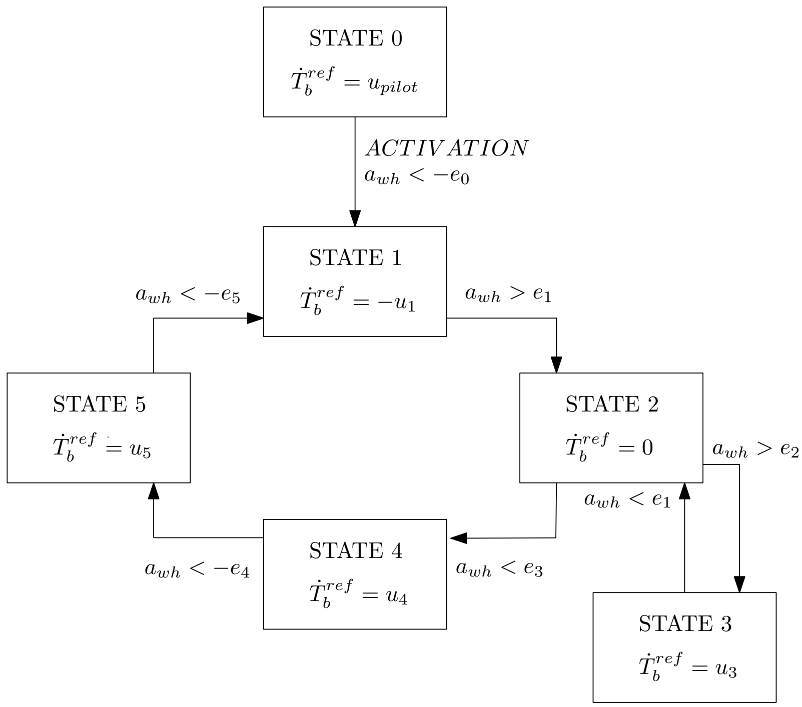


Fig. 5 Finite state machine representing the basic version of the 5-phase braking control algorithm in [12].

4 Structure of the anti-skid controllers

In order to study the interplay between tire wear and anti-skid control, we need to introduce the two braking control approaches that will be used in the following. Specifically, the model of a deceleration-based controller that induces a limit-cycle behaviour on the wheel skid is first introduced, followed by another approach that directly controls the wheel skid.

4.1 Traditional anti-skid control approach

For studying the behaviour of commercial aeronautic anti-skid controllers, in view of the specification of using a single measurement given by the rotational speed of the wheel of the considered landing gear, a deceleration-based control algorithm must be employed.

For the design of such an algorithm we leveraged on the work first presented in [12], where a threshold-based control algorithm was proposed, using only the wheel deceleration, that allows achieving a stable limit cycle on the wheel slip, which in principle should position itself around the optimal value of the wheel slip for the given friction condition, of course assumed to be unknown. Such an algorithm is referred to as *5-phase* in view of its finite state machine representation that is shown in Fig. 5. As can be seen, the algorithm, based on thresholds on the wheel deceleration a_{wh}, computed by appropriately differentiating the measured wheel speed, imposes a value to the time derivative of the braking torque $\dot{T}_b$, which can be either zero, or positive or negative. According to its value, the torque in the given phase of the algorithm will be held constant, or increased, or decreased. The alternation of such actions allows obtaining a limit cycle of the wheel slip which, with the given basic version of the algorithm, is proved to enjoy stability properties.

When such an algorithm is to be implemented in a real system, where actuator dynamics and measurement noises must be taken into account, the basic version shown in Fig. 5 must be modified, in order to maintain the desired features. The interested reader is referred to [5] for more details.

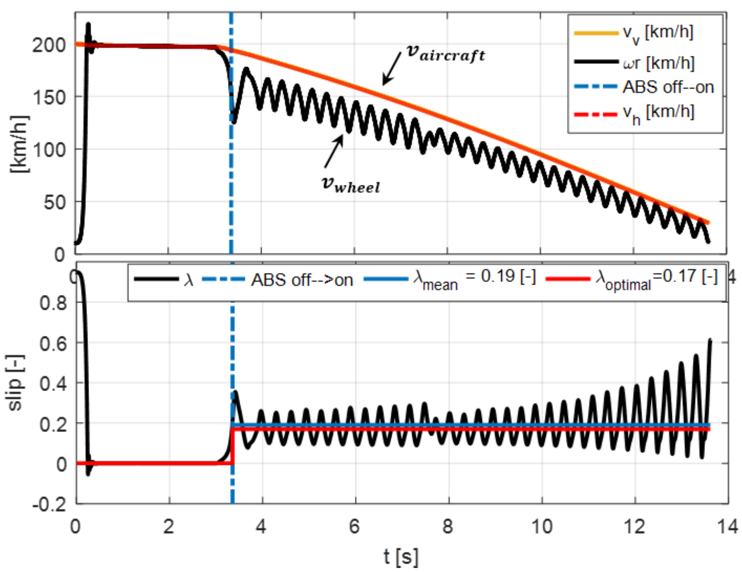


Fig. 6 Closed-loop behaviour of the 5-phases anti-skid control on dry road. Top plot: time histories of aircraft and wheel speed; bottom plot: time histories of the wheel slip.

A typical closed-loop behaviour of such controller is depicted in Figure 6, which shows that the wheel slip indeed evolves on a limit-cycle, and that such a cycle is almost centered around the (unknown) optimal value of the slip for the given road condition, dry road in the case show in the figure. By computing the average wheel slip achieved in the braking maneuver, indicated with λ_{mean} in the figure. Note that λ_{mean} will be the variable of interest to define the tire-wear associated to the braking maneuver. In the case shown in Fig. 6 $\lambda_{mean} = 0.19$ was obtained .

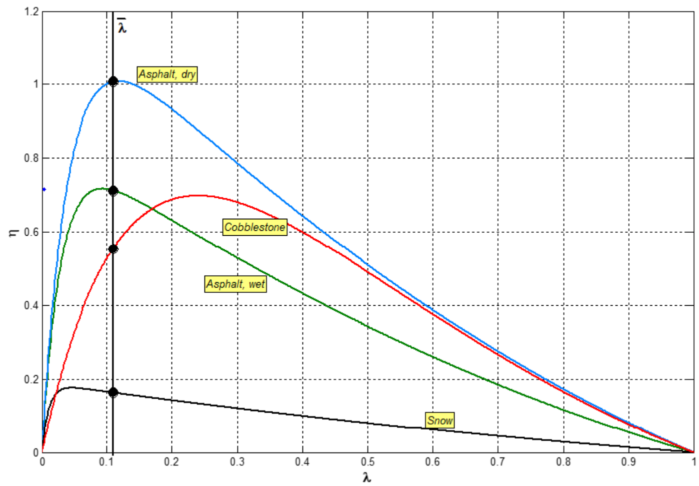


Fig. 7 Pictorial representation of the slip-control algorithm in the $(\lambda, \eta$ domain.

4.2 Slip control

It is now possible to introduce the slip controller designed for anti-skid purposes. To do this, we first define the normalized wheel deceleration as

$$\eta = -\frac{\dot{\omega} r}{g}, \tag{11}$$

where $\dot{\omega}$ is the wheel deceleration and g is the gravitational acceleration. Such a variable allows one to compare directly the wheel deceleration to that of the body of the aircraft expressed in g. In a linearized context, it is possible to pictorially describe the slip control principle in the (λ, η) domain using the representation shown in Fig. 7. This figure shows, for each considered road condition, the (η, λ) equilibrium manifolds obtained from the linearized equations of the single corner model. The vertical solid line defines the set-point value of the wheel slip to be tracked by the slip control system. As can be seen, a single closed-loop equilibrium point exists for each road surface and for each choice of the set-point value, and a good performance trade-off can be obtained with a fixed choice of such a set-point value on all friction conditions. The controller, in this case, is designed based on the linearized braking dynamics with standard frequency-domain loop-shaping methods for linear and time-invariant systems, leading to a transfer function representation $R(s)$ of the final controller, [15].

A typical closed-loop behaviour of the slip controller on dry road is shown in Fig. 8 proving that the wheel slip indeed evolves toward the desired set-point value, $\lambda_{ref} = 0.17$ in the case depicted in the figure. The following section will relate the

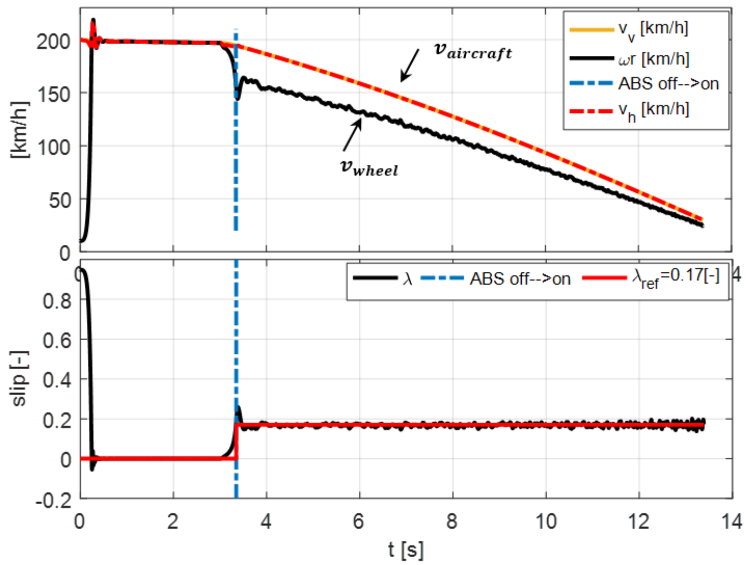


Fig. 8 Closed-loop behaviour of the slip control system on dry road. Top plot: time histories of aircraft and wheel speed; bottom plot: time histories of the wheel slip.

wheel slip set-point value, assuming it will be reached and maintained during the maneuver thanks to the action of the slip controller, to the tire consumption.

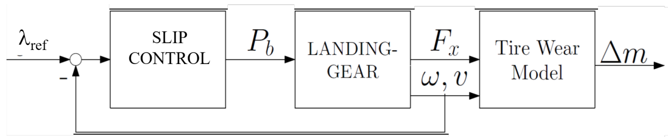
5 Tire-wear analysis of anti-skid braking

Based upon the modelling of the tire wear phenomenon described in Section 3, it is now possible to evaluate and compare the tire mass-loss obtained with both anti-skid control logics described in Section 4. To concisely describe the performance side of the control system, the stopping distance ΔS is considered as a relevant and easily interpretable cost function.

Considering the models in Fig.s 9a and 9b, the procedure used to compute the tire mass-loss in a closed-loop braking maneuver is shown for the two control approaches.

To explore the trade-off between performance, expressed in terms of stopping distance ΔS, and tire mass-loss Δm, different values of the wheel slip set-point have been considered for the slip control algorithm. Consider that such a tuning is not possible for the 5-phase control system, as its paradigm is that of not specifying a wheel slip set-point, but to automatically bring the wheel slip of the closed-loop system to cycle around its optimal value, *i.e.*, that corresponding to the peak of the friction curve for the current runaway condition.

(a) From anti-skid control to mass-loss computation: 5-phase algorithm



(b) From anti-skid control to mass-loss computation: slip-control algorithm

Fig. 9 Closed-loop ant-skid control with computation of the tire consumption in terms of mass-loss Δm.

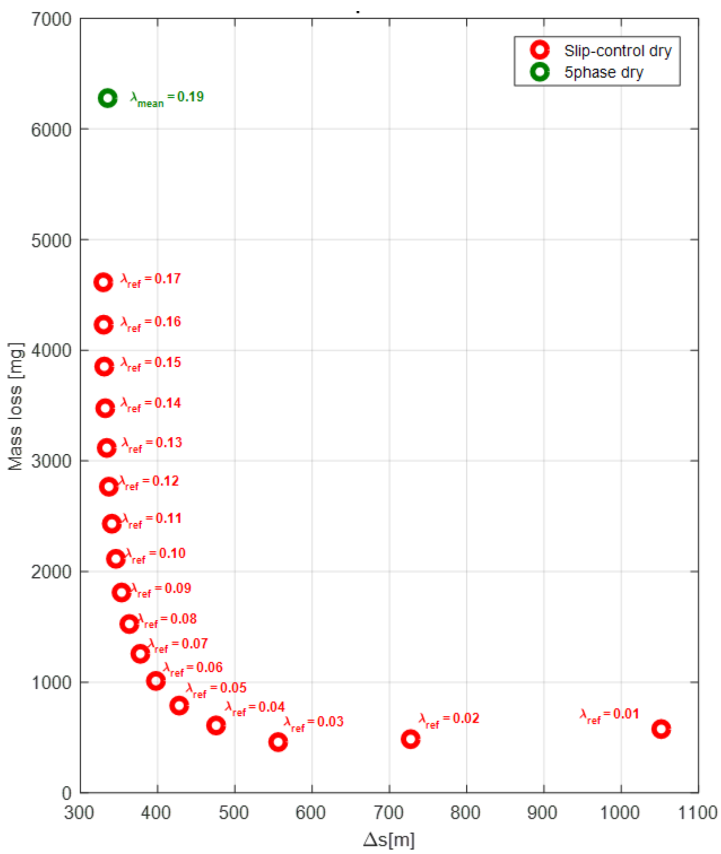


Fig. 10 Pareto curve of tire mass-loss versus stopping distance in closed-loop anti-skid braking.

Visualizing the performance-consumption trade-off in the stopping distance vs. mass-loss plane, the results shown in Fig. 10 are obtained. As can be seen, by varying the wheel slip set-point a huge variation in tire consumption can be achieved. Most interestingly, a strong and tangible difference between slip-based and acceleration-based control approaches is revealed: looking at the results achieved with the 5-phase algorithms, in fact, one may see that absolutely reasonable and acceptable results are obtained as far as performance are concerned, but little or no degrees of freedom are available to act on the tire consumption.

In fact, by tuning the controller parameters small modifications of the resulting limit cycle may be obtained, but these in turn reflect in only very small variations of the average value of the wheel slip obtained in closed-loop, and thus minor modifications of the final tire mass-loss. Note also that such modifications can be carried out having in mind not to alter the closed-loop stability and robustness against, for example, the unknown road conditions.

Looking at the slip control approach, instead, it is apparent that a great flexibility in accommodating constraints that can optimally mix performance and tire consumption is indeed available. In fact, thanks to the shape of the resulting Pareto curve in Fig. 10, large savings in tire mass can be obtained at the price of very small degradation in terms of stopping distance.

This concept keeps unaltered if one computes the Pareto curve on other road conditions. As an example, Fig. 11 shows the comparison of the results obtained on dry and wet runaways. As can be seen, the same rationale applies. Moreover, note that slip-control can be proved to offer closed-loop stability and robustness against all road conditions for any choice of the set-point value λ_{ref}.

Overall, these results show that a very promising way to be able of directly and knowingly influencing the tire consumption in aircraft is to design the anti-skid control system solving a slip regulation problem. Of course, this implies that a new measurement must be made available to obtain an estimate of the aircraft speed. If connecting the main flight controller with the braking system to directly send to it the aircraft speed measurement given by the main inertial measurement unit (IMU), a landing-gear-based solution can be devised placing a local IMU on the rigid part of the landing gear to get information for estimating the aircraft speed, thus avoiding critical coupling among the different subsystems.

6 Concluding Remarks

This paper considered the problem of tire consumption in aircraft braking, offering a model that allows computing the mass-loss experienced by a tire during anti-skid braking. Furthermore, the interplay between closed-loop braking control and tire consumption has been studied, revealing an interested opportunity to strongly reduce tire consumption at the price of a small, and *a priori* predictable, loss of braking performance, using an anti-skid control approach designed solving a slip regulation problem. These results favorably witness the effectiveness of the approach, and in

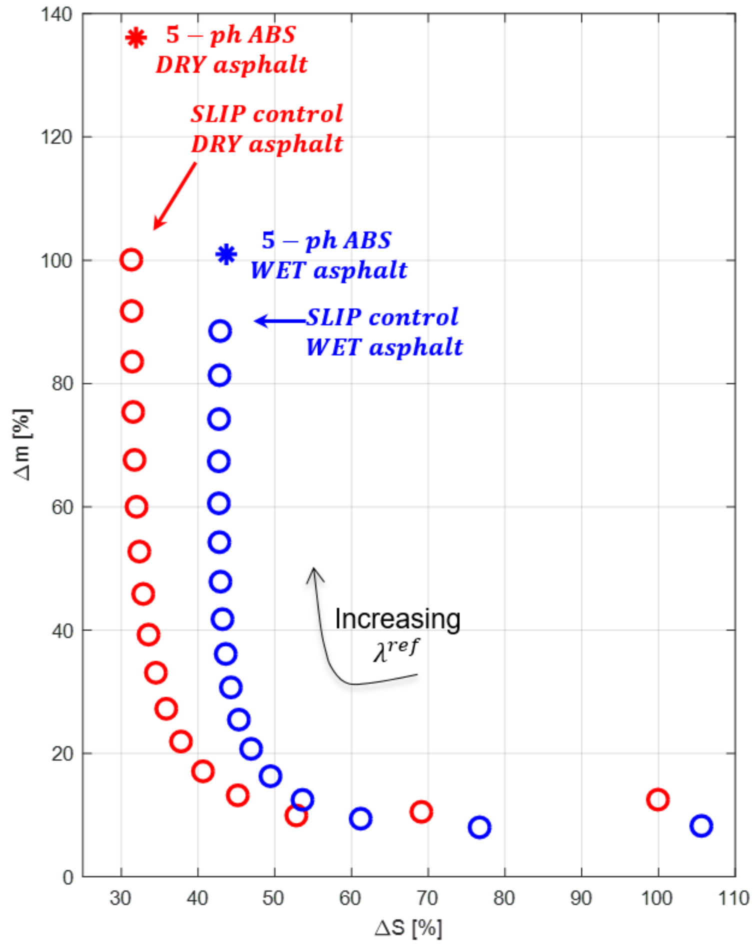


Fig. 11 Pareto curve of tire mass-loss versus stopping distance in closed-loop anti-skid braking.

our view strongly support the start of a new generation of anti-skid controllers for aircraft that make use of an additional measurement to estimate the aircraft speed that would allow to compute the wheel slip. Ongoing work is being devoted to the experimental validation of this idea.

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